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WHAT IS CLAIMED IS:

A method for recovering 3D scene structure and camera motion from image data obtained from a multi-image sequence, wherein a reference image of the sequence is taken by

- a camera at a reference perspective and one or more successive images of the sequence are taken at one or more successive different perspectives by translating and/or rotating the camera, the method comprising the steps of:
 - (a) determining image data shifts for each successive image with respect to the reference image; the shifts being derived from the camera translation and/or rotation from the reference perspective to the successive different perspectives;
 - (b) constructing a shift data matrix that incorporates the image data shifts for each image;
 - (c) calculating two rank-3 factor matrices from the shift data matrix using SVD, one rank-3 factor matrix corresponding to the 3D structure and the other rank-3 factor matrix corresponding to the camera motion;
 - (d) recovering the 3D structure from the 3D structure matrix by solving a linear equation; and
 - (e) recovering the camera motion from the camera motion matrix using the recovered 3D structure.

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The method of claim 1, wherein the image data is one or more selected from the 2. NECI1071

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group consisting of points, lines and intensities.

- 3. The method of claim 1, wherein the step of determining image data shifts includes initially recovering and compensating for camera rotation.
- The method of claim 1\(\) wherein step (b) comprises: 4.

computing H and \overline{D}_{CH} , where H is a $(N_{tot} - 3) \times N_{tot}$ matrix and $\overline{D}_{CH} \equiv C^{-1/2} \overline{D} H^T$ and $N_{\text{tot}} \equiv 2N_p + 2 N_L + N_X$, where N_p , N_L , and N_X equal the number of points, lines and intensities, respectively, and C is a $(N_1-1) \times (N_1-1)$ matrix with $C_{ii'} \equiv \delta_{ii'} + 1$ where $C_{ii'}$ is a constant table of values for all images and $\delta_{ii'}$ is an operator equal to 1 if i = i' and 0 if $i \neq i'$,

and $\overline{D} = [S \omega_1 \wedge \omega_1 \Delta]$, H^T is the transpose of the H matrix, where H is a matrix defined such that \mathbf{H}^{T} is an identity matrix and annihilates $\overline{\Psi}_{x}$, $\overline{\Psi}_{y}$, $\overline{\Psi}_{z}$ where $\overline{\Psi}_{x}^{T} \equiv \left[\Psi_{x}^{T} \omega_{\mathrm{L}} \Psi_{Lx}^{T} \omega_{\mathrm{I}} \Psi_{Lx}^{T}\right]$ and similarly for the y and z components where, ω_1 and ω_L are constant weights that the user

sets, and where
$$\Psi_{Lx} = \begin{bmatrix} \{ P_U \cdot (\hat{\mathbf{x}} \times \mathbf{A}) \} \\ \{ P_L \cdot (\hat{\mathbf{x}} \times \mathbf{A}) \} \end{bmatrix}, \Psi_{Ly} = \begin{bmatrix} \{ P_U \cdot (\hat{\mathbf{y}} \times \mathbf{A}) \} \\ \{ P_L \cdot (\hat{\mathbf{y}} \times \mathbf{A}) \} \end{bmatrix}, \Psi_{Lz} = \begin{bmatrix} \{ P_U \cdot (\hat{\mathbf{z}} \times \mathbf{A}) \} \\ \{ P_L \cdot (\hat{\mathbf{z}} \times \mathbf{A}) \} \end{bmatrix}$$
 where A_l is

the unit normal to the plane containing the line and the camera center at the reference image and where \hat{x} , \hat{y} , \hat{z} are unit vectors in the x, y and z directions, and where

$$\Psi_{x} = \begin{bmatrix} \left\{ r_{x}^{(1)}(q) \right\} \\ \left\{ r_{y}^{(1)}(q) \right\} \end{bmatrix}, \quad \Psi_{y} = \begin{bmatrix} \left\{ r_{x}^{(2)}(q) \right\} \\ \left\{ r_{y}^{(2)}(q) \right\} \end{bmatrix}, \quad \Psi_{z} = \begin{bmatrix} \left\{ r_{x}^{(3)}(q) \right\} \\ \left\{ r_{y}^{(3)}(q) \right\} \end{bmatrix} \text{ where } q = (x,y) \text{ is the image position of the tracked point in the reference image and}$$

- the three point rotational flows $r^{(1)}(x,y)$, $r^{(2)}(x,y)$, $r^{(3)}(x,y)$ are defined by 20 $[r^{(1)}, r^{(1)}, r^{(1)}] \equiv \left[\begin{pmatrix} -xy \\ -(1+y^2) \end{pmatrix}, \begin{pmatrix} 1+x^2 \\ xy \end{pmatrix}, \begin{pmatrix} -y \\ x \end{pmatrix} \right]$ and where $\Psi_{lx} = -\{\nabla I \cdot \mathbf{r}^{(1)}(\mathbf{p})\}, \Psi_{ly} = -\{\nabla I \cdot \mathbf{r}^{(2)}(\mathbf{p})\}, \Psi_{lz} = -\{\nabla I \cdot \mathbf{r}^{(3)}(\mathbf{p})\}, \text{ and where } p \text{ gives the image}$ coordinates of the pixel positions, and
- where Λ is an $N_i \times 2N_i$ matrix where each row corresponds to a different image i and 25 equals $\left\{P_U \cdot \delta A^i\right\}^T \left\{P_L \cdot \delta A^i\right\}^T$ where P_U and P_L are unit 3-vectors projecting onto two directions $A_1 \times (\hat{z} \times A_1)$ and $\hat{z} \times A_1$ which we refer to respectively as the upper and lower g\nec\1196\13701\spec\13701pj1 **NECI1071** 32

directions, where δA^i is the line flow $\delta A^i_I \equiv A^i_I - A^0_I$, and \hat{z} is the unit vector in the z direction and where S is a $N \times 2N_p$ matrix where each row corresponds to a different image i and equals $\left[\left\{ s^i_x \right\}^T \left\{ s^i_y \right\}^T \right]$, where $s^i_m \equiv q^i_m - q^0_m$ and denotes the image displacement for the m-th tracked point and where Δ is a $N_I \times N_X$ matrix, where each row corresponds to an image i and equals $\left\{ \Delta I^i \right\}^T$ where ΔI is the change in image intensity with respect to the reference image and where I^i denotes the i-th intensity image and where $I^i_n = I^i(p_n)$ denotes the image intensity at the n-th pixel position in I^i and where the notation $\{V\}$ is used to denote a vector with elements given by the V^a .

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- The method of claim 1, wherein step (c) comprises:

 computing the best rank-3 factorization of $\overline{D}_{CH} \approx M^{(3)} S^{(3)T}$ where $M^{(3)}$, $S^{(3)}$ are rank a matrices corresponding respectively to motion and structure, using SVD.
- 15 6. The method of claim 1, wherein step (d) comprises:

eliminating structure unknowns Q_z , B_z and Z^{-1} from the $\overline{\Phi}_a$ to get $3N_{tot}$ linear constraints on the U and Ω using the linear equation, $[\overline{\Phi}_x \overline{\Phi}_y \overline{\Phi}_z] = H^T S^{(3)} U + [\overline{\Psi}_x \overline{\Psi}_y \overline{\Psi}_z] \Omega$, where U and Ω are unknown 3x3 matrices, and $\overline{\Phi}$ and $\overline{\Psi}$ represent total translational and rotational flow vectors respectively, and solving these constraints with $O(N_{tot})$ computations using the

SVD, where the total translational flow vectors are defined by $\overline{\Phi}_x \equiv \begin{bmatrix} \Phi_x \\ \omega_L \Phi_{Lx} \\ \omega_l \Phi_{lx} \end{bmatrix}$ and similarly

for the y and z components, and

$$\Phi_{Lx} = \begin{bmatrix} \{A_x B_U \} \\ \{A_x B_L \} \end{bmatrix}, \Phi_{Ly} = \begin{bmatrix} \{A_y B_U \} \\ \{A_y B_L \} \end{bmatrix}, \Phi_{Lz} = \begin{bmatrix} \{A_z B_U \} \\ \{A_z B_L \} \end{bmatrix}$$
 where A is the normal to a first plane that

passes through the center of projection and the imaged line in the reference image, and the normal to a second plane, B is defined by requiring $B \cdot A = 0$ and $B \cdot Q = -1$ for any point Q the 3D line L, and where $B_U \equiv B \cdot P_U$ and $B_L \equiv B \cdot P_L$ are the upper and lower components of B, and where

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$$\Phi_{x} \equiv -\begin{bmatrix} Q_{z}^{-1} \\ \{0\} \end{bmatrix}, \Phi_{y} \equiv -\begin{bmatrix} \{0\} \\ \{Q_{z}^{-1} \} \end{bmatrix}, \Phi_{z} \equiv -\begin{bmatrix} \{q_{x}Q_{z}^{-1} \} \\ \{q_{y}Q_{z}^{-1} \} \end{bmatrix}$$

and where

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$$\Phi_{Ix} \equiv -\left\{Z^{-1}I_x\right\}, \Phi_{Iy} \equiv -\left\{Z^{-1}I_y\right\}, \Phi_{Iz} \equiv \left\{Z^{-1}(\nabla I \cdot \mathbf{p})\right\}, \text{ and}$$

where Q is the 3d coordinate for a 3D tracked point corresponding to an image pixel in the reference image, and Z_n is the depth from the camera to the 3D point imaged at the n-th pixel along the cameras optical axis; and

recovering the structure unknowns Q_z , B_z and Z^{-1} from

$$\left[\overline{\Phi}_x \overline{\Phi}_x \overline{\Phi}_x\right] = H^T S^{(3)} U + \left[\overline{\Psi}_x \overline{\Psi}_y \overline{\Psi}_z\right] \Omega$$
, given U and Ω .

7. The method of claim 1, wherein step (e) comprises:

using
$$S^{(3)}U \approx \left[\overline{\Phi}_x \overline{\Phi}_y \overline{\Phi}_z\right]$$
 and

15 $\overline{D}_{CH} \approx C^{-\frac{1}{2}} \{T_x\} \overline{\Phi}^T H^T + C^{-\frac{1}{2}} \{T_y\} \overline{\Phi}_y^T H^T + C^{-\frac{1}{2}} \{T_z\} \overline{\Phi}_z^T H^T$ to recover the translations, and recovering the rotations $\omega_x^i, \omega_y^i, \omega_z^i$ from

$$\omega_x^i \overline{\Psi}_{xn} + \omega_{yx}^i \overline{\Psi}_{yn} + \omega_{zx}^i \overline{\Psi}_{zn} = C^{-\frac{1}{2}} \overline{D}_n^i - \left(C^{-\frac{1}{2}} (T_x) \overline{\Phi}_y^T + \{T_y\} \overline{\Phi}_y^T + \{T_z\} \overline{\Phi}_z^T \right)_n^i, \text{ wherein } C \text{ is a constant } (N_1 - 1) \text{ x } (N_1 - 1) \text{ matrix with } C_{ii'} = \delta_{ii'} + 1 \text{ and } T \text{ represents the translation.}$$

8. A method for recovering 3D scene structure and camera motion from image data obtained from a multi-image sequence, wherein a reference image of the sequence is taken by

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13701.pje a camera at a reference perspective and one or more successive images of the sequence are taken at one or more successive different perspectives by translating and/or rotating the camera, the method comprising the steps of:

- (a) initially recovering and compensating for camera rotation after warping the reference image by the previously estimated translational displacements;
 - (a) determining image data shifts for each successive image with respect to the reference image; the shifts being derived from the camera translation and/or rotation from the reference perspective to the successive different perspectives;
- (b) constructing a shift data matrix that incorporates the image data shifts for each image;
- (c) modifying the image shifts δA_l^i , s_m^i , ΔI_n^i and redefining these quantities to include the denominator factors $1 - Q_z^{-1}T_z$ for points, $1 - Z^{-1}T_z$ for intensities and $1 - B \cdot T$ for lines where $s_m^i \equiv q_m^i - q_m^0$ and denotes the image displacement for the *m*-th tracked point, where Δ_I^i is the change in image intensity and where I^i denotes the *i*-th intensity image and where $I_n^i = I^i(p_n)$ denotes the image intensity at the *n*-th pixel position in I^i and where δA_l^i represents the line flow
- (d) calculating two rank-3 factor matrices from the modified shift data matrix using SVD, one rank-3 factor matrix; corresponding to the 3D structure and the other rank-3 factor matrix corresponding to the camera motion;
- (e) recovering the 3D structure from the 3D structure matrix by solving a linear **NECI1071** 35



- (f) recovering the camera motion from the camera motion matrix using the recovered 3D structure.
- The method of claim 8, wherein the image data is one or more selected from the 5 9. group consisting of points, lines and intensities.



10. The method of claim 8, wherein step (b) comprises:

computing H and \overline{D}_{CH} , where H\sigma is a $(N_{tot} - 3) \times N_{tot}$ matrix and $\overline{D}_{CH} \equiv C^{-1/2} \overline{D} H^T$ and $N_{\text{tot}} \equiv 2N_p + 2 N_L + N_X$, where N_p , N_L , and N_X equals the number of points, lines and intensities, respectively, and C is a $(N_1-1)(x (N_1-1))$ matrix with $C_{ii'} \equiv \delta_{ii'} + 1$ where $C_{ii'}$ is a constant table of values for all images and $\delta_{ii'}$ is an operator equal to 1 if i = i' and 0 if $i \neq i'$

and $\overline{D} = [S \omega_L \Lambda \omega_1 \Delta]$, H^T is the transpose of the H matrix, where H is a matrix defined such that H^T is an identity matrix and annihilates $\overline{\Psi}_x, \overline{\Psi}_y, \overline{\Psi}_z$ where $\overline{\Psi}_x^T = \left[\Psi_x^T \omega_L \Psi_{Lx}^T \omega_I \Psi_{Lx}^T \right]$ and similarly for the y and z components and where ω_1 and ω_L are constant weights that the user sets, and where $\Psi_{Lx} \equiv \begin{bmatrix} \{P_U \cdot (\hat{\mathbf{x}} \times \mathbf{A})\} \\ \{P_L \cdot (\hat{\mathbf{x}} \times \mathbf{A})\} \end{bmatrix}, \Psi_L \equiv \begin{bmatrix} \{P_U \cdot (\hat{\mathbf{y}} \times \mathbf{A})\} \\ \{P_L \cdot (\hat{\mathbf{y}} \times \mathbf{A})\} \end{bmatrix}, \Psi_{Lz} \equiv \begin{bmatrix} \{P_U \cdot (\hat{\mathbf{z}} \times \mathbf{A})\} \\ \{P_L \cdot (\hat{\mathbf{z}} \times \mathbf{A})\} \end{bmatrix}$ where

 A_l is the unit normal to the plane containing the line L and the camera center at the reference image, and where $\hat{x}, \hat{y}, \hat{z}$ are unit vectors in the x, y and z directions, and where

$$\Psi_{x} = \begin{bmatrix} \left\{ r_{x}^{(1)}(q) \right\} \\ \left\{ r_{y}^{(1)}(q) \right\} \end{bmatrix}, \ \Psi_{y} = \begin{bmatrix} \left\{ r_{x}^{(2)}(q) \right\} \\ \left\{ r_{y}^{(2)}(q) \right\} \end{bmatrix}, \ \Psi_{z} = \begin{bmatrix} \left\{ r_{x}^{(3)}(q) \right\} \\ \left\{ r_{y}^{(3)}(q) \right\} \end{bmatrix}$$
 where $q = (x, y)$ is the image position of the

tracked point in the reference image and 20

the three point rotational flows $r^{(1)}(x, y), r^{(2)}(x, y), r^{(3)}(x, y)$ are defined by $\left[r^{(1)}, r^{(1)}, r^{(1)}\right] \equiv \left[\left(\frac{-xy}{-(1+y^2)}\right), \left(\frac{1+x^2}{xy}\right), \left(\frac{-y}{x}\right)\right]$ and where

$$[r^{(1)}, r^{(1)}, r^{(1)}] = [(-xy - (1+y^2)), (xy - (1+x^2)), (xy - y)]$$

 $\Psi_{lx} = -\left\{\nabla I \cdot \mathbf{r}^{(1)}(\mathbf{p})\right\}, \Psi_{ly} = -\left\{\nabla I \cdot \mathbf{r}^{(2)}(\mathbf{p})\right\}, \Psi_{lz} = -\left\{\nabla I \cdot \mathbf{r}^{(3)}(\mathbf{p})\right\}, \text{ and where } p \text{ gives the image}$ coordinates of the pixel positions, and

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where Λ is an $N_I \times 2N_L$ matrix where each row corresponds to a different image i and equals $\left[\left\{ \mathbf{P}_U \cdot \delta \mathbf{A}^i \right\}^T \left\{ \mathbf{P}_L \cdot \delta \mathbf{A}^i \right\}^T \right]$ where \mathbf{P}_U and \mathbf{P}_L are a unit 3-vector projecting onto two directions $\mathbf{A}_I \times (\hat{z} \times \mathbf{A}_I)$ and $\hat{z} \times \mathbf{A}_I$ which we refer to respectively as the upper and lower directions, where $\delta \mathbf{A}^i$ is the line flow $\delta \mathbf{A}_I^i \equiv \mathbf{A}_I^i - \mathbf{A}_I^0$, and \hat{z} is the unit vector in the z direction and where S is a $N_i \times 2N_p$ matrix where each row corresponds to a different image i and equals $\left[\left\{ \mathbf{s}_x^i \right\}^T \left\{ \mathbf{s}_y^i \right\}^T \right]$, where $\mathbf{s}_m^i \equiv \mathbf{q}_m^i - \mathbf{q}_m^0$ and denotes the image displacement for the m-th tracked point and where Δi is a $N_I \times N_X$ matrix, where each row corresponds to an image i and equals $\left\{ \Delta I^i \right\}^T$ where ΔI is the change in image intensity with respect to the reference image and where I^i denotes the i-th intensity image and where $I^i = I^i(\mathbf{p}_n)$ denotes the image intensity at the n-th pixel position in I^i and where the notation $\{V\}$ is used to denote a vector with elements given by the V^a .

11. The method of claim 8, wherein step (d) comprises:

computing the best rank-3 factorization of $\overline{D}_{CH} \approx M^{(3)} S^{(3)T}$ where $M^{(3)}$, $S^{(3)}$ are rank 3 matrices corresponding respectively to motion and structure, using SVD.

12. The method of claim 8, wherein step (e) comprises:

eliminating structure unknowns Q_z , B_z and Z^{-1} from the $\overline{\Phi}_a$ to get $3N_{tot}$ linear constraints on the U and Ω using the linear equation, $[\overline{\Phi}_x \overline{\Phi}_y \overline{\Phi}_z] = H^T S^{(3)} U + [\overline{\Psi}_x \overline{\Psi}_y \overline{\Psi}_z] \Omega$, where U and Ω are unknown 3x3 matrices, and $\overline{\Phi}$ and $\overline{\Psi}$ represent total translational and rotational flow vectors respectively, and solving these constraints with $O(N_{tot})$ computations using the

SVD, where the total translational flow vectors are defined by $\overline{\Phi}_x \equiv \begin{bmatrix} \Phi_x \\ \omega_L \Phi_{Lx} \\ \omega_l \Phi_{lx} \end{bmatrix}$ and similarly for the y and z components, and

 $\Phi_{Lx} = \begin{bmatrix} \{A_x B_U \} \\ \{A_x B_L \} \end{bmatrix}, \Phi_{Ly} = \begin{bmatrix} \{A_y B_U \} \\ \{A_y B_L \} \end{bmatrix}, \Phi_{Lz} = \begin{bmatrix} \{A_z B_U \} \\ \{A_z B_L \} \end{bmatrix}$ where A is the normal to a first plane that passes through the center of projection and the imaged line in the reference image, and the NECI1071

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normal to a second plane, B is defined by requiring $B \cdot A = 0$ and $B \cdot Q = -1$ for any point Q on the 3D line L, and where $B_U \equiv B \cdot P_L$ and $B_L \equiv B \cdot P_L$ are the upper and lower components of B, and where

$$\Phi_{x} = -\begin{bmatrix} \left\{ Q_{z}^{-1} \right\} \\ \left\{ 0 \right\} \end{bmatrix}, \Phi_{y} = -\begin{bmatrix} \left\{ 0 \right\} \\ \left\{ Q_{z}^{-1} \right\} \end{bmatrix}, \Phi_{z} = -\begin{bmatrix} \left\{ q_{x} Q_{z}^{-1} \right\} \\ \left\{ q_{y} Q_{z}^{-1} \right\} \end{bmatrix}$$

and where

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$$\Phi_{Ix} \equiv -\{Z^{-1}I_x\}, \Phi_{Iy} \equiv -\{Z^{-1}I_y\}, \Phi_{Iz} \equiv \{Z^{-1}(\nabla I \cdot \mathbf{p})\}, \text{ and}$$

where Q is the 3d coordinate for a tracked 3D point in the reference image, and Z_n is the depth from the camera to the 3D point imaged at the n-th pixel along the cameras optical axis; and

recovering the structure unknowns Q_z , B_z and Z^{-1} from

$$\left[\overline{\Phi}_x \overline{\Phi}_x \overline{\Phi}_x\right] = H^T S^{(3)} U + \left[\overline{\Psi}_x \overline{\Psi}_y \overline{\Psi}_z\right] \Omega$$
, given U and Ω .

13. The method of claim 8, wherein step (f) comprises:

using
$$S^{(3)}U \approx \left[\overline{\Phi}_x \overline{\Phi}_v \overline{\Phi}_z\right]$$
 and

 $\overline{D}_{CH} \approx C^{-\frac{1}{2}} \{T_x\} \overline{\Phi}^T H^T + C^{-\frac{1}{2}} \{T_y\} \overline{\Phi}_y^T H^T + C^{-\frac{1}{2}} \{T_z\} \overline{\Phi}_z^T H^T \text{ to recover the translations, and recovering the rotations } \omega_x^i, \omega_y^i, \omega_z^i \text{ from}$

$$\omega_{x}^{i}\overline{\Psi}_{xn} + \omega_{yx}^{i}\overline{\Psi}_{yn} + \omega_{zx}^{i}\overline{\Psi}_{zn} = C^{-\frac{1}{2}}\overline{D}_{n}^{i} - \left(C^{-\frac{1}{2}}(\{T_{x}\}\overline{\Phi}^{T} + \{T_{y}\}\overline{\Phi}_{y}^{T} + \{T_{z}\}\overline{\Phi}_{z}^{T})\right)_{n}^{i}, \text{ wherein } C \text{ is a}$$

$$\text{constant } (N_{1}-1) \text{ x } (N_{1}-1) \text{ matrix } \text{ with } C_{ii'} \equiv \delta_{ii'} + 1 \text{ and } T \text{ represents the translation.}$$

